Quiz 14

FALL 2022 FINAL EXAM

12. (a) (2 points) What is the 3rd degree Taylor Polynomial, $T_3(x)$, of $\sin(x)$ centered at $\pi/2$?

    (b) (1 point) Approximate $\sin(1)$ using $T_3(x)$. You do not need to simplify your answer.

    (c) (3 points) Use Taylor's Inequality to give an upper bound for the error in this approximation. You do not need to simplify your answer.

15. Let $f(x) = x^4 \arctan(x^3)$.

    (a) (3 points) Use the known Taylor series for $\arctan(x)$ centered at 0 to find the Taylor series for $f(x)$ centered at 0.

    (b) (3 points) Use your response to (a) to find $f^{(19)}(0)$.

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9. (3 points) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{9^n (2n)!}$.

    A. 0

    B. $\frac{1}{2}$

    C. $\cos\left(\frac{\pi}{9}\right)$

    D. $\sin\left(\frac{\pi}{9}\right)$

    E. $\sqrt{3}$

10. (3 points) Consider the function $f(x) = \sin(x^3)$. What is the 15th derivative of f evaluated at 0? That is, what is $f^{(15)}(0)$?

    A. $f^{(15)}(0) = \frac{1!}{15!}$

    B. $f^{(15)}(0) = \frac{1}{5!}$

    C. $f^{(15)}(0) = 0$

    D. $f^{(15)}(0) = \frac{15!}{5!}$

    E. $f^{(15)}(0) = \frac{5!}{15!}$

16. (10 points) Let $T_n(x)$ be the Taylor polynomial of degree n for a function $f(x)$ centered at $a = 4$. Suppose $\left|f^{(n+1)}(x)\right| \leq 8$ for all x and $n \geq 0$. Use Taylor's Inequality to find how large n must be for $T_n(5)$ to approximate $f(5)$ with an error $|R_n(5)| \leq 0.1$.

Quiz 14

Multiple Choice

1. Use the known Taylor series for e^x / sin(x) / cos(x) / arctan(x) / ln(1+x) / etc. to find the Taylor series for f(x), centered at a = 0. For example, f(x) = x^4arctan(x^3)

2. Calculate the exact sum of a given series.

3. Let T_n(x) be the Taylor polynomial of degree n for a function f(x) centered at a = 4. Suppose |f^(n+1)(x)| ≤ 8 for all n ≥ 0. Use Taylor's Inequality to find how large n must be for T_n(5) to approximate f(5) with an error |R_n(x)| ≤ 1. (I'll change the numbers)

Free Response

(a) What is the 3rd degree Taylor polynomial of sin(x) centered at a = ? (I might do π/2, π/3, or π/6)

(b) Use Taylor's Inequality to give an upper bound for the error in this approximation. You do not need to simplify your answer.

(have memorized)

What you need to know:

Common Maclaurin Series

The following are standard Maclaurin series for important functions. You are expected to memorize these and be able to use them to generate new series by substitution, differentiation, or integration.

Function	Maclaurin Series	Radius of Convergence
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	∞
$\sin x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	∞
$\cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	∞
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	1
$\ln(1+x)$	$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$	1
$\arctan x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$	1

Theorem (Taylor's Inequality)**.** If $\left|f^{(n+1)}(x)\right| \leq M$ for $|x-a| \leq d$, then the remainder $R_n(x)$ of the Taylor series satisfies

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \quad \text{for } |x-a| \leq d.$$